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FRACTAL MODELS IN PROBLEMS OF AUTOMATIC CONTROL THEORY

Аннотация*: в данной статье рассмотрены вопросы сложности не предсказуемых поведений физических систем, включая систему автоматического управления. Представлены методы статистического анализа случайных величин и статистического анализа функций.*

Ключевые слова*: автоматизация, контроль, рандомизация, анализ, характеристики.*

Abstract*: in this article the complexity of unpredictable behaviours of physical systems, including the automatic control system, is considered. Methods of statistical analysis of random variables, and statistical analysis of functions.*

Keywords*: automation, control, randomization, analysis, characteristics.*

As is well known complex unpredictable behavior of a physical system including automatic control system may be due to random changes in its parameters, random external influences, as well as the development of various instabilities in the system. These factors lead to randomization of signals and structures that characterize the behavior and state of the system. To study the processes often involved randomization of various probabilistic approaches.

At the heart of these approaches are methods of statistical analysis of random variables and statistical analysis of random variables and functions. Along with them in recent years have become widespread and some less well-known signal processing techniques, based in particular on the fractal, multi- fractal analysis and wavelet transform. A distinctive feature of the past is that they, along with the global characteristics of stochastic processes allow features of their local structure.

An important characteristic of the method is based on fractal representations is their versatility. They are used to study a wide range of complex irregular phenomena in the natural and in the humanities. We will be mainly interested in those options are techniques that are most relevant to the specific problems of the theory of automatic control.

First we give a general description of fractals. Let's start with a definition of fractals.

Fractals called geometric objects: line, surface, spatial body with a very jagged shape and having the property of self-similarity. Fundamental theory of fractals B. Mandelbrot [1] the term fractal is formed from the Latin participle fractious, which means to break, crush, crush, i.e. create fragments of irregular shape.

Self-similarity as the main characteristic of a fractal means that it is more or less uniformly arranged in a wide range of scales. Thus, an increase in small fragments fractal obtained very similar to the big ones. This determines the scale invariance (scaling) of the basic geometric features of a fractal object, their invariance under change of scale.

Fractal property is determined by its fractal dimension D. It is defined by Mandelbrot to the following formula:

$$D = -\lim_{I \to 0} \frac{in\ N(I)}{in\ I} \tag{1}$$

Where N (I) – the number of crushable shapes (sphere, cube, square, etc.)

I – the size of the figures.

The value of D is a local characteristic of the object.

Let the control object is subject to random perturbations and measurement of the controlled variable done with some noise. Assume that both of these random variables (functions) have a Gaussian distribution, i.e., the output signal has a Gaussian signal.

It is considered that if the signal (output) $Y(t)$ with a standard deviation σ is subject to model generalized Brownian motion if the increment.

$$\Delta Y = Y(t_2) - Y(t_1) \tag{2}$$

has a Gaussian distribution, characterized by expressions

$$P(\Delta y < y) = \frac{1}{\sqrt{2\pi\sigma(t_2 - t_1)H}} \int_{-\infty}^{Y} \exp\left[-\frac{1}{2}\left(\frac{U}{\sigma(t_2 - t_1)^H}\right)^2\right] dU \tag{3}$$

(3) that the model HBS delta – the dispersion is

$$M\left[\left(Y(t_2) - Y(t_1)\right)^2\right] = \sigma^2 \left|t_2 - t_1^{\ 2H}\right| \tag{4}$$

Included in the above ratio parameter $H(o < H < 1)$ is a parameter of Hurst.

The expectation of the increment of the output signal (manipulated variable), or even the so-called first-order structure function is given by

$$M\left[\left|Y(t_2) - Y(t_1)\right|\right] = \sqrt{\frac{2}{\pi}\sigma(t_2 - t_1)^H} \tag{5}$$

Increments have the property of self-similarity statistic, which is expressed mathematically as follows:

$$Y(t + \Delta t) - Y(t) \cong \frac{2}{2^H}(Y(t + r\Delta t) - Y(t)) \tag{6}$$

For any $r > 0$

The fractal dimension of the output signal is calculated by the formula (1), i.e.,

$$D = -\lim_{\Delta t \to 0} \frac{\log N(\Delta t)}{\log \Delta t} = 2 - H. \tag{7}$$

Simulation output fractal signal for the entire range of possible changes in the fractal dimension $1 < D < 2$ can be carried out with the help of the Weierstrass function [2; 3]

$$Y(t) = \sqrt{2\sigma}\, \frac{\left[1 - b^{2D-4}\right]^{\frac{1}{2}} \sum_{n=0}^{N} b^{(D-2)n} \cos\left(2\pi S b^n t + \psi_n\right)}{\left[1 - b^{(2D-4)(N+1)}\right.} \tag{8}$$

where – the standard deviation, b, S-parameters space – frequency scaling, D-fractal dimension associated with Hurst parameter value for $D = 2 - H$. $N + 1$ is the number of harmonics – phase distributed randomly in the interval, t – time.

Us to simulate the output signal $Y(t)$ are the graphs of model signals for the values of the fractal dimension and. Graphs of the output signals were constructed using the Weierstrass functions (8).

Building structure functions showed that the output signals are described using the Weierstrass function are fractal. Hurst parameters were found to be $H = 0{,}91$ and $H = 0{,}34$. The corresponding fractal dimension equal $D_1 = 1{,}12t \pm 0{,}02$ and $D_2 = 1{,}78t \pm 0{,}03$ Average dimensions are very close initially set values $D_1 = 1{,}1$ and $D_2 = 1{,}8$ asked when building dependencies Y_k^1 and Y_k^2.

References

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