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ФОРМУЛА ДЛЯ ОПРЕДЕЛЕНИЯ ПРОГИБА РЕШЕТЧАТОЙ ФЕРМЫ С ПРОИЗВОЛЬНЫМ ЧИСЛОМ ПАНЕЛЕЙ

***Аннотация**: автором данной статьи получена формула для оценки жесткости плоской внешне статически неопределимой балочной фермы. Для нахождения общего решения использован метод индукции и операторы системы компьютерной математики Maple. Усилия в стержнях найдены методом вырезания узлов, прогиб – по формуле Максвелла – Мора. Выведены также формула для горизонтального смещения опоры. Найдены некоторые асимптотические свойства решения.*

***Ключевые слова**: ферма, прогиб, точное решение, Maple, индукция.*

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THE FORMULA FOR DETERMINING THE DEFLECTION OF THE TRUSS WITH AN ARBITRARY NUMBER OF PANELS

***Abstract**: the author has obtained a formula for the evaluation of hardness flat externally, statically indeterminate girders. To obtain the General solution the researcher uses the method of induction and the operators of the computer algebra system Maple. The forces in the rods was found by using cut nodes method, deflection – using Maxwell – Mohr integral. The article has also derived the formula for the horizontal displacement of the support. Some asymptotic properties of the solution are found.*

***Keywords**: truss, deflection, an exact solution, Maple, induction.*

Derive formula for deflection of a girder (Fig. 1) under the action of a concentrated force at Midspan. Symmetrical truss includes $2n$ panels and $m = 8\ (n + 1)$ rods (including four supports). The feature of this design is that it is impossible to determine all support reactions of the usual equilibrium equations for the entire truss as a whole.

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However, the truss is statically determinate. In truss 4 (n + 1) of joints. The condition of equilibrium of them enables to find the forces in all rods, including the supports. For the derivation of the dependence of the deflection of the number of panels it is necessary to obtain a number of formulae for trusses with different numbers of panels, and then by induction to find a General formula. A similar technique was previously used in problems on the flat [1–14] and spatial trusses [8; 9]. The calculation of the optimum height of the truss is also possible in an approximate Kachurin's formula [10].

The forces in the rods will determine by the method of cutting of knots. Using Maxwell – Mohr integral

$$\Delta = P\sum_{i=1}^{m-4} S_i^2 l_i / (EF),$$

where S_i the forces in the rods of the truss under the action of external loads, l_i – the lengths of the rods, EF is the stiffness of the rods adopted the same for the whole structure. A series of analytical solutions obtained in the system of computer mathematics Maple program given in [11] for different number of panels generalize by induction, making the recurrence equation for the coefficients of the required formulas and solving them. In the process of counting, it was observed that for even number of panels in half span, the determinant of the system of equations becomes zero. The calculations produced for n = 2k-1.

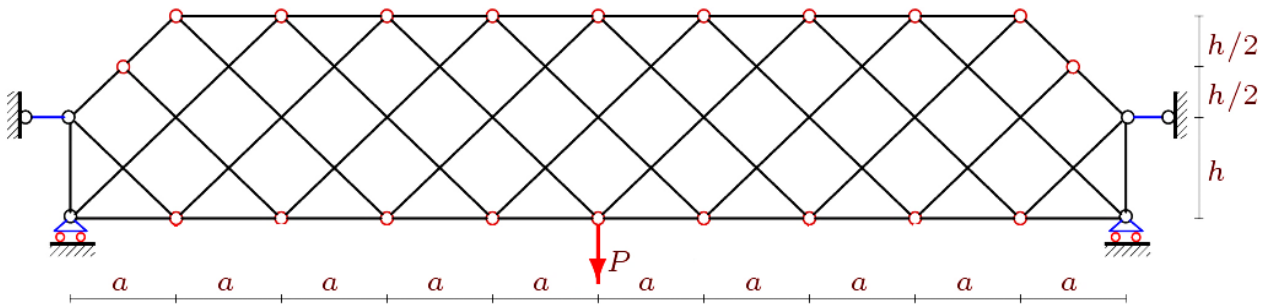

Fig. 1. Truss, k = 3

We give the fragment of program of the input the coordinates of the nodes:

> *mu*: = *4*n+8*: # *The number of nodes with supports*

Enter the coordinates of the nodes of the bottom chord:

> *for i to 2*n + 1 do x*[*i*]: = *a**(*i-1*); *y*[*i*]: = *0*; *od*:

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> *x*[*2*n +2*]: = *0*:*y*[*2*n + 2*]: = *h*:*x*[*4*n + 4*]: = *2*n*a*:*y*[*4*n + 4*]: = *h*:

> *x*[*2*n + 3*]: = *a*/*2*:*y*[*2*n + 3*]: = *3*h*/*2*:*x*[*4*n+3*]: = *2*n*a-a*/*2*:*y*[*4*n + 3*]: = *3*h*/*2*:

Top chord:

> *for i to 2*n-1 do x*[*I + 2*n + 3*]: = *a*i*; *y*[*I + 2*n + 3*]: = *2*h*; *od*:

Enter the coordinates of supported nodes:

> *x*[*mu-3*]: = *-2*: *y*[*mu-3*]: = *h*:

> *x*[*mu-2*]: = *0*: *y*[*mu-2*]: = *-4*:

> *x*[*mu-1*]: = *x*[*2*n + 1*]: *y*[*mu-1*]: = *-4*:

> *x*[*mu*]: = *x*[*2*n + 1*] + *4*: *y*[*mu*]: = *h*:

To enter the structure of the truss, you must specify the order of connection of terminals (rods) and nodes. This function is performed by special vectors *N*[*i*] consisting of vertices at the ends of the rods. The vectors of the lower belt are of the form

> *for i to 2*n do N*[*i*]: = [*i*,*i+1*];*od*:

The upper zone (top chord) correspond to the vectors

> *for i to 2*n + 2 do N*[*I + 2*n*]: = [*i + 2*n + 1*,*i + 2*n + 2*];*od*:

The rods of the lattice are introduced in the following way

> *for i to 2*n do N*[*i + 4*n + 2*]:=[*i + 1*,*i + 2*n + 1*];

> *N*[*i + 6*n + 2*]:=[*i + 2*n + 4*,*i*];*od*:

> *N*[*8*n + 3*]: = [*1*,*2*n + 2*]: *N*[*8*n + 4*]: = [*2*n + 1*,*4*n + 4*]:

Consistent calculation of trusses with k = 1,2, 3... 12 shows that the formula for the deflection of the Central node of the bottom chord has every time the same form

$$EF\Delta_k = P\frac{A_k a^3 + C_k c^3 + h^3}{2h^2},$$

where $c = \sqrt{a^2 + h^2}$. According to the obtained results by induction from the solution of the recurrent equations of the 5th order

$$A_k \ = \ 3A_{k-1} - 2A_{k-2} - 2A_{k-3} + 3A_{k-4} - A_{k-5},$$

we derive the following expressions for the coefficients:

$$A_k = (4/3)k^3 - 2k^2 + (8/3)k + (-1)^k,\ C_k = 2k - 1.$$

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An analogous formula for the horizontal displacement of the vertical support under load $EF\Delta_h = P\dfrac{A_k a^3 + C_k c^3 + h^3}{2n^2a^2}$,

where $A_k = (2/3)n(10k^2 - 10k + 3),\ C_k = n(2k^2 - 2k + 1).$

You can get formulas for the horizontal lateral reactions of the supports, the sign of which turns out to be depends on the parity of the number of panels $S_{hor} = (-1)^{k+1}Pa/(2h)$. The vertical reactions are obvious and does not depend on the number of panels $S_{vert} = P/2$.

Graph of relative deflection for a given span length L=2an=40m is given in figure 2, the offset – in figure 3. It is interesting to note that for small heights of the curves of deflection of the panels have an extremum (minimum). In addition, these curves have points of intersection, which indicates the ambiguity of the decision, allowing to optimize the construction. The offset of support (Fig. 3) grows with the number of panels.

Operators of Maple allow us to find some asymptotic properties of solutions for a fixed span length. The limit $\lim\limits_{k\to\infty}\dfrac{\Delta'}{k} = \dfrac{h}{L}$ shows that the curves in figure 2 have an oblique asymptote, and the growth displacement is cubic in nature

$$\lim_{k\to\infty}\frac{\Delta_h'}{k^3} = 6\left(\frac{h}{L}\right)^3.$$

In [12] gives a comparative review of analytical solutions of the deflection of some flat trusses.

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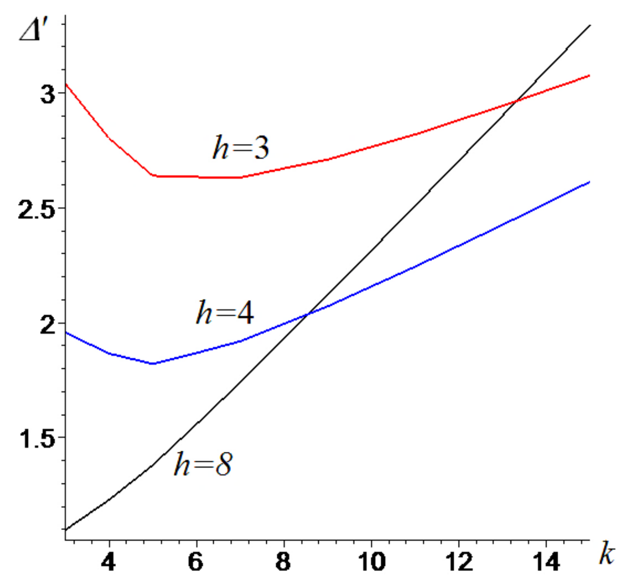

Fig. 2. The deflection of the truss

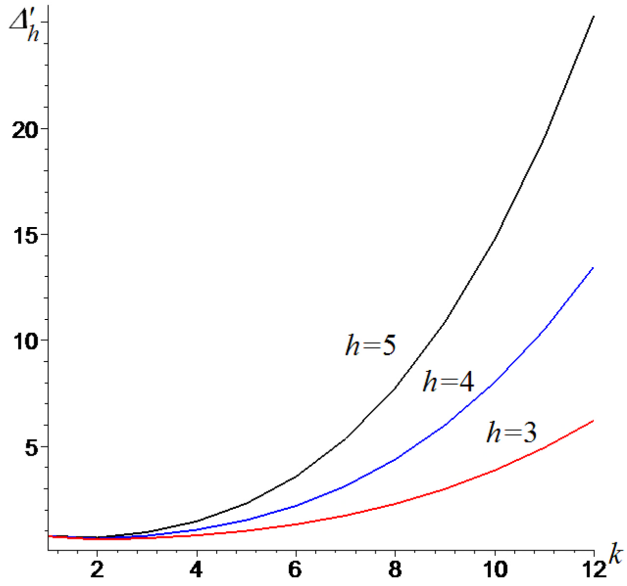

Fig. 3. Offset of the support

Compared to Kachurin' formula, a main feature of the solution is its accuracy and consideration of the work of a particular lattice. Almost without any significant changes the algorithm can be used in other systems of computer mathematics system: Mathematica, Maxima etc.

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