***Павлов Андрей Валерианович***

канд. физ.-мат. наук, доцент

ФГБОУ ВПО «Московский государственный технический университет радиотехники, электроники и автоматики»

г. Москва

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# ПРЕОБРАЗОВАНИЯ ЛАПЛАСА, ОРТОГОНАЛЬНЫЕ ПРЕОБРАЗОВАНИЯ, ПОЛЯ СДВИГОВ

***Аннотация**: в статье доказано, что с точки зрения чисел и нового скалярного произведения диагонали произвольного ромба можно считать одинаковыми как результат ортогонального преобразования, переводящего стороны в диагонали в случае, когда длины векторов измеряются в одних и тех же единицах измерения длин на сторонах. Во второй части статьи приведен некоторый класс функций, значения которых восстанавливают только по известным положительным значениям преобразований Лапласа од данных функций. В третьей части статьи приведены примеры, когда отображения точек комплексной плоскости становятся периодичными с произвольным периодом независимо от исходного аналитического отображения (с точки зрения введения двух систем координат).*

***Ключевые слова**: оптимальное линейное приближение, ортогональные преобразования, обращение преобразования Лапласа, возникновение периодичности аналитических функций.*

**Pavlov Andrey Valerianovich**

cand. ph.-math. science  associate professor,

FSBEI of HE “MIREA – Russian Technological University”

Moscow

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# THE TRANSFORM OF LAPLACE, ORTHOGONAL TRANSFORMATIONS, MOVING FIELDS

***Abstract**: it is proved in article, that from point of numbers and new scalar work of diagonal of arbitrary rhombus it is possible to consider identical as a result of the orthogonal transformation, when lengths of vectors are measured in the same units of measuring as on sides of the rhombus. In the second part of article some class of functions is resulted: the values of the functions restore on the known positive values of the transform of Laplace. In the third part of article the examples are resulted, when a function of points of complex plane become periodic with the arbitrary period (from point of some introduction of two systems of co-ordinates).*

***Keywords**: orthogonal transformations, linear projection, transform of Laplace, moving fields.*

*Introduction*.

The optimal linear

$x_3$

projection of the $x_3$ vector to the linear subspace of the $x_1, x_2$ vectors (on the plane of the $x_1, x_2$ vectors, $(x_1, x_2) \neq 0$) can be written in the form

$$x_3 = c_1 x_1 + c_2 x_2, x_3 - (c_1 x_1 + c_2 x_2) \perp x_i, i = 1, 2,$$

[1], or in the form

$$x_3 = (e_1, x_3)e_1 + (e_2, x_3)e_2,\ e_1 \perp e_2, \| e_1 \| = \| e_2 \| = 1, x_3 - x_3 \perp e_i, i = 1, 2.$$

A purpose of the article is some proof (model validation) of the fact: from point of a numerical values the

$$x_3 = C_1 x_1 + C_2 x_2$$

equality with the

$$C_i = (x_{n+1}, x_i) / \| x_i \|^2$$

coefficients is possible, if

$$(x_1, x_2) \neq 0,\ i = 1, 2, n = 2$$

(from point of geometrical images the expression is impossible, if $(x_1, x_2) \neq 0$).

We will use a *two facts*, [*1–8*], (the facts are interesting from point of algebra. In the proposition 1 we consider the new scalar production in the linear subspace of 2 vectors $(x_1, x_2)_2$. With help of the new production we obtain a new form of the linear

$x_3$

projection of the additional $x_3$ vector to the $x_1, x_2$ subspace. As usually we determine two lengths of vectors

$\| x \|_i = \sqrt{(x,x)_i}, i = 1,2; (x,y)_1 = (x,y)$.

From point of the linear independence of vectors the projection must have the same length as for the initial first metrics:

$\| x_3 \|_1 = \| x_3 \|_2$

(the proposition 1, but

$\| x_3 \|_1 \neq \| x_3 \|_2$).

The elementary proposition is an introduction to the basic result of article in the lemma 1. In the proposition 1 we use the linear independence of the

$x_3$

projection and the

$x_3 - x_3$

vector in both metrics

($x_3 - x_3 \perp x_3$).

The fact conflicts with the definition of length for the second metrics. It is necessary to enter some new units of length on the

$x_3$

line, but the length are the same for both metrics on the sides of parallelogram with the $x_1, x_2$ sides. *It is the first fact*. From point of the fact or the

$x_3 = c_1 x_1 + c_2 x_2,$

equality (the new form of the optimal linear projection) takes place with

$c_i = (x_{n+1}, x_i) / \| x_i \|^2, i = 1,2, n = 2,$

or the notion of linear dependence is violated.

The introduction of new unit of vectors for the second metrics in proposition 1 is impossible from point of some orthogonal $A_1$ transformation on plane (the lemma 1) too. *It is the second interesting fact of the article*.

We use, that a diagonals of rhombus with the $x_1, x_2, \| x_1 \| = \| x_2 \|$, sides are orthogonal for both scalar productions. As the result of the orthogonal linear $A_1$ transformations of the $Q_1, Q_2$ diagonals we obtain the orthogonal $x_1 = AQ_1, x_2 = AQ_2$ vectors (for both scalar productions). We obtain, that in the symmetric situation ( $x_3$ is the diagonal ) it is possible to suppose, that the $x_1, x_2$ vector are orthogonal, and the above new algorithm of finding of optimal linear projection of the $x_3$ vector on the subspace of $x_1, x_2$ vectors ensues from the projection as for $(x_1, x_2) = 0$ so as for $(x_1, x_2) \neq 0$.

*1. Some result about the scalar productions*.

In the lemma 1 we use the definitions of the [8] article.

*Lemma 1*.

$$(A_1 y_1, A_1 y_2)_2 = (A_1 y_1, A_1 y_2)_1$$

for all the $y_1 \in L, y_2 \in Lj$ vectors from point of digital presentation of both scalar productions and lengthes.

*Proof*.

We obtain

$$(A_1 y_1, A_1 y_2)_2 = (A_1(\alpha_1 e_1 + \beta_1 e_2), A_1(\alpha_2 e_1 + \beta_2 e_2))_2 =$$

$$= ((\alpha_1 A_1 e_1 + \beta_1 A_1 e_2), (\alpha_2 A_1 e_1 + \beta_2 A_1 e_2))_2 = \alpha_1 \alpha_2 (A_1 e_1, A_1 e_1)_2 + \beta_1 \beta_2 (A_1 e_2, A_1 e_2)_2 = J,$$

(we use

$$(A_1 e_i, A_1 e_j) = (A_1 e_i, A_1 e_j)_2 = 0$$

for both scalar productions, if $i \neq j$,

with help of

$$(A_1 e_i, A_1 e_i)_2 = 1,$$

if

$$(e_i, e_i)_2 = 1, i = 1, 2$$

as the result of the orthogonal transformation of the vectors,

$(e_1, e_2)_2 = 0, i = 1, 2$

, we get

$(A_1 e_i, A_1 e_i) = 1, i = 1, 2$

- it is the same length 1 as on the sides of the rhombus with help of the equalities

$(A_1 e_1, A_1 e_1)_2 = (e_1, e_1)_2 = 1,$

and

$(e_i, e_i)_1 = (e_i, e_i)_2 = 1, i = 1, 2$

from point of digital presentation of both scalar productions. In the situation we compare the lengths on diagonal with the lengths on sides in the numeral expression for both metrics (the length for the second metric is equal to the d units of measuring of the units on the sides of rhombus and the d values are the same for both metrics). If to make to use notion of parallel, after the turn of picture of rhombus the length of the turned side is measured in the same units of measuring as length of initial diagonal, if the initial diagonal parallel to the turned side.

We get

$J = \alpha_1 \alpha_2 (A_1 e_1, A_1 e_1) + \beta_1 \beta_2 (A_1 e_2, A_1 e_2) =$

$= ((\alpha_1 A_1 e_1 + \beta_1 A_1 e_2), (\alpha_2 A_1 e_1 + \beta_2 A_1 e_2))_1 = (A_1 y_1, A_1 y_2)_1,$

$\alpha_1 e_1 + \beta_1 e_2 = y_1, \alpha_2 e_1 + \beta_2 e_2 = y_2, \quad \alpha_i, \beta_i = const., e_i = x_i / \| x_i \|_1 = x_i / \| x_i \|_2, \| e_i \|_i = 1, i = 1, 2.$

We use

$A_1 e_1 = (e_1 + e_2)/\sqrt{2}, A_1 e_2 = (e_1 - e_2)/\sqrt{2}, \quad \| x_1 + x_2 \|_2 = \| x_1 - x_2 \|_2$.

The lemma 1 is proved.

As the result of the lemma 1 we get

$(x_1, x_2) = (A_1 Q_1, A_1 Q_2) = 0,$

but

$(x_1, x_2) \neq 0$

as a primary assumption. With help of the proposition 1 and lemma 1 the fact is possible (for an usual assumptions) from point of digital presentation of both scalar productions and lengths.

*2. The transform of Laplace.*

In the first part we use the lemma of Jordan.

By definition,

$$LZ(t)(\cdot)(x) = \int_0^\infty e^{-xt} Z(t)dt, x \in [0,\infty),$$

$$F_\pm u(t)(\cdot)(p) = \int_{-\infty}^\infty e^{\pm pit} u(t)dt, \quad p \in (-\infty,\infty),$$

$$Cu(t)(\cdot)(x) = \int_{-\infty}^\infty u(t)\cos xtdt, \quad x \in (-\infty,\infty).$$

*Theorem 1.*

1.

$$LF_+[1/(t+\lambda+i\beta)](\cdot)(p) = 2\pi i/(p+\lambda i-\beta), \beta < 0,$$

$$LF_-[1/(t+\lambda-i\beta)](\cdot)(p) = 0, \beta < 0;$$

$$LF_+[1/(t+\lambda+i\beta)](\cdot)(p) = 0, \beta > 0,$$

$$LF_-[1/(t+\lambda+i\beta)](\cdot)(p) = \overline{2\pi i/(p+\lambda i+\beta)}, \beta > 0,$$

for all

$Re\, p > -|\beta|$.

2.

$$LC[1/(t+\lambda+i\beta)](\cdot)(p) = \pi i/(p+\lambda i-\beta) = \pi u(-ip),$$

$$u(t) = 1/(t+\lambda+i\beta), \beta < 0; \quad LC[1/(t+\lambda+i\beta)](\cdot)(p) = \pi(-i)/(p-\lambda i+\beta) = \pi u_1(ip), \beta > 0,$$

for all

$Re\, p > -|\beta|$, $u_1(p) = 1/(t+\lambda+i\beta)$.

It is obviously, the proof we can obtain from the

$$\int_{a-i\infty}^{a+i\infty} [e^{sit}/p]dp = I(t)$$

equality (page 464, [5],Lavrentiev,Shabat, Methods of the complex analysis), where

$$I(t) = 1, t \in (0,\infty), I(t) = 0, t \in (-\infty,0), a > 0.$$

*Theorem 2*.

1.

$$\pi i/(p+\lambda i-\beta)=\int_0^\infty e^{-px}dx\int_0^\infty \cos itx[(1/(t+\lambda+i\beta))+(1/(-t+\lambda+i\beta))]]dt=$$

$$=\int_0^\infty \cos pxdx\int_0^\infty e-tx[2(\lambda+i\beta)/(\lambda+i\beta)^2-t^2)]dt, \beta<0, p\in[0,\infty),$$

2.

$$\pi(-i)/(p-\lambda i+\beta)=$$

$$=\int_0^\infty \cos pxdx\int_0^\infty e-tx[2(\lambda+i\beta)/(\lambda+i\beta)^2-t^2)]dt, \beta>0, p\in[0,\infty).$$

As result of the theorem 2 we obtain the remark 1.

*Remark 1*.

$$\int_0^\infty \cos xtf(t)dx=(\pi/2)\int_0^\infty e^{-tx}u(t)dt, t\in[0,\infty),$$

where

$$u(t)=\sum_{j=1}^{n}[(1/(t+\lambda_j+i\beta_j))+(1/(-t+\lambda_j+i\beta_j))], t\in[0,\infty),$$

with main the condition $u(0)=0,$ and the $f(t)$ is the appropriate sum

$$\sum_{j=1}^{n}(\pm i)/(p\pm\lambda_j i\mp\beta_j),$$

for all

$$\lambda_j,\beta_j\in(-\infty,0)\bigcup(0,\infty), j=1,\ldots,, n\in 1,2,\ldots.$$

In of principle plan the task of the u(t) search in the $Lu(t)(\cdot)(x)$ expression only by the positive values is decided for the wide class of functions of the theorems 1 and 2 (for the even functions).

*3. Periodic functions*.

The theorem 3 continues the results of reasons [8] in relation to the origin of periodicity of functions on plane. If to consider the values of analytical functions of f(r-2A) in points with the first A coordinate, we will get the «two moving fields»

F(p): F(p)=f(p-2A),Re p=A;

G(p): G(p)=f(p-2Ai),Im p=A ,

(from point of analytical continuations the «moving field» in some conditions is equal to the initial function, the theorem 3, [8]).

The

z = g(p) = f(2A - p)

function is equal to the symmetric reflection of values of the f(p) function in relation to the (A,0) point (we use

g(p) = f(A - (p - A),

if

f(A + (p -A)) = f(p) ,

or the complex z=f(p) function). By definition, it is the A-reflection. In the theorem 3 we consider some results similar to the [8] work. It is usefully to mark also a fact ): for the A-reflection we obtain a two f(3A-w) and g(3A-p) equations in two systems of co-ordinates with centers on the axis OX (in the distance A from each other, w=p+A); it is the two equations of the only (z,f(2A-p)) set of points ( the set for the primary system of coordinates ). In the initial co-ordinates (the p variable ) we reflect the f(p) function in the p=(A,0) point, and in the second co-ordinates (the w variable ) we reflect the g(w) function in the w= (2A,0) point. We can consider the parallel 3A - w and 3A-p vectors. The (z, f(3A-w)) and (z,g(3A-p)) sets are moved one in relation to other on the 2A distance. The values of the parallel vectors moved one in relation to other on the A distance. A beginning of the vectors are located in the centers of co-ordinates. We proved, that the initial z=f(Z) function (as the comparison of points of plane)) has become periodic with the A>0 period. The fact is the basic result of article (it is reported on the DFDE2022 conference in Moscow).

*Theorem 3*.

For all

-4A<Re p<4A>0,

in some system of co- ordinates the

$Z = f(X + iY), X = y, Y = x,$

«moving field» is equal to the g(X+iY) regular function (in relation to the X+iY=W complex variable), [8].

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